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ARREST OF A MOVING MASS BY AN ATTACHED MEMBRANE

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SUMMARY

Large-deflection solutions are obtained for a fixed-end membrane strip and for a fixed circular membrane, each with a centrally located attached mass initially moving at a given speed. The solutions are given in the form of equations and curves for the deflections and stresses.

INTRODUCTION

For certain space-flight applications it is desirable to have a structure that can be packaged for launching and that can be deployed in space by inflation. Structures which satisfy such requirements are shells with membranelike walls. The attachment of concentrated masses to such membranelike structures may be required and during the inflation process these masses introduce dynamic loads into the structure. In order to design such a structure to survive deployment, it is important to be able to calculate the stresses and deformations in a membrane structure due to the motion of an attached mass. To obtain a realistic distribution of stresses, large-deflection (nonlinear) theory is essential. In this paper large-deflection solutions are obtained in closed form for two fundamental problems involving the arrest of a centrally located moving mass attached to a membrane. A centrally located mass is considered to be attached to (1) a membrane strip fixed at its ends or (2) a circular membrane fixed at its outer circumference. The assumption is made that the mass of the membrane is small in comparison with the attached mass. For both mass-membrane systems the results for the deflections and stresses as a function of time are given in equations and curves.

SYMBOLS

a	half-length or radius of mass (fig. 1)
b	half-length or outside radius of membrane (fig. 1)
C	constant of integration
E	Young's modulus of membrane material

f	characteristic speed distribution
g_1, g_2	functions of time
h	thickness of membrane
M	mass
m	mass per unit width
N	longitudinal or radial stress resultant (N_x or N_r)
N_r, N_θ	direct stress resultants in polar coordinates
N_x	longitudinal stress resultant
r	radial distance
t	time
t_A	time of arrest of mass
u	displacement in x- or r-direction
V_o	initial speed of mass
w	deflection normal to initial plane of membrane
x	longitudinal distance from center of strip (fig. 1)
μ	Poisson's ratio of membrane material

When the subscripts x, r, and t follow a comma, they indicate partial differentiation of the principal symbol with respect to x, r, and t, respectively.

MEMBRANE-STRIP ANALYSIS

The first configuration considered is a fixed-end membrane strip of length 2b and any desired width. Attached to it is a centrally located mass of length 2a and of (mass) intensity m per unit width. (See fig. 1(a).) Only deflections symmetric about the center line are considered. The boundary conditions on displacements are zero deflection at $x = b$:

$$w(b,t) = 0 \tag{1}$$

and zero in-plane displacement at $x = a$ and $x = b$:

$$\left.\begin{aligned} u(a,t) &= 0 \\ u(b,t) &= 0 \end{aligned}\right\} \tag{2}$$

The boundary condition on stress resultant at the mass (at $x = a$) is

$$N_x(a,t)w_{,x}(a,t) = \frac{m}{2}w_{,tt}(a,t) \tag{3}$$

Initially the strip-mass system is assumed to have zero lateral deflection and some characteristic speed distribution with the mass moving at speed V_o. Thus the initial conditions are

$$w(x,0) = 0 \tag{4}$$

and

$$w_{,t}(x,0) = f(x) \tag{5}$$

where the characteristic speed distribution $f(x)$ must be equal to zero at $x = b$ and equal to V_o at $x = a$.

The equation of equilibrium of the longitudinal forces in the membrane strip is

$$N_{x,x} = 0 \tag{6}$$

The stress resultant N_x is given in terms of displacements by

$$N_x = \frac{Eh}{1 - \mu^2}\left(u_{,x} + \frac{1}{2}w_{,x}{}^2\right) \tag{7}$$

From equation (6) it is seen that N_x is independent of x. With the mass of the membrane considered negligible in comparison with the attached mass, the equation of equilibrium of the transverse forces is

$$\left(N_x w_{,x}\right)_{,x} = 0 \tag{8}$$

which, since N_x is independent of x, requires that w be linear in x:

$$w = g_1(t) + xg_2(t) \tag{9}$$

It then follows from equations (2) and (7) that $u = 0$ everywhere. Condition (1) gives $g_2(t) = -\frac{1}{b}g_1(t)$, or

$$w = \left(1 - \frac{x}{b}\right)g_1(t) \tag{10}$$

The characteristic speed distribution is, therefore,

$$f(x) = V_o \frac{b - x}{b - a} \tag{11}$$

Substitution of equation (10) into equations (3), (4), and (5) gives the following nonlinear differential equation and initial conditions from which g_1 may be found:

$$\ddot{g}_1 + \frac{Eh}{b^2(b - a)m\left(1 - \mu^2\right)}g_1^3 = 0 \tag{12}$$

$$g_1(0) = 0 \tag{13}$$

$$\dot{g}_1(0) = V_o \frac{b}{b - a} \tag{14}$$

Equation (12) is multiplied through by $\dot{g}_1$ and integrated to get

$$\dot{g}_1 = \sqrt{C - \frac{Eh}{2b^2(b - a)m\left(1 - \mu^2\right)}g_1^4} \tag{15}$$

where C, the constant of integration, may be determined from equations (13) and (14) to be

$$C = \frac{b^2V_o^2}{(b - a)^2}$$

Equation (15) has the following solution:

$$g_1 = b\sqrt[4]{\frac{2\left(1 - \mu^2\right)mV_o^2}{(b - a)Eh}}\,\mathrm{cn}\left[\sqrt[4]{\frac{2EhV_o^2}{(b - a)^3\left(1 - \mu^2\right)m}}\left(t_A - t\right),\frac{\sqrt{2}}{2}\right] \tag{16}$$

where cn is the elliptic cosine and t_A is the time of arrest, which is found in the tables of elliptic functions to be

$$t_A = 1.8541\left(1 - \frac{a}{b}\right)^{3/4} \sqrt[4]{\frac{(1 - \mu^2)mb^3}{2EhV_o^2}} \tag{17}$$

to satisfy condition (13).

The deflection and stress resultant can now be written

$$\frac{w}{b} = \frac{1 - \frac{x}{b}}{\left(1 - \frac{a}{b}\right)^{1/4}} \sqrt[4]{\frac{2(1 - \mu^2)MV_o^2}{Ehb}} \operatorname{cn}\left[1.8541\left(1 - \frac{t}{t_A}\right), \frac{\sqrt{2}}{2}\right] \tag{18}$$

and

$$N_x = \frac{1}{\left(1 - \frac{a}{b}\right)^{1/2}} \sqrt{\frac{EhmV_o^2}{2(1 - \mu^2)b}} \operatorname{cn}^2\left[1.8541\left(1 - \frac{t}{t_A}\right), \frac{\sqrt{2}}{2}\right] \tag{19}$$

Note that the stress is a function of time only. The results given in equations (17) to (19) are also presented in figures 2, 3, 4, 5, and 6.

Consider now the case in which the membrane strip is initially slack so that the mass moving at speed V_o does not draw the membrane tight until the slope of the membrane is $-\beta$. Initial condition (4) is changed to

$$w(x,0) = \beta(b - x)$$

Condition (13) is accordingly

$$g_1(0) = \beta b \tag{20}$$

and C is given by

$$C = \frac{b^2}{(b - a)^2}\left[V_o^2 + \frac{Eh\beta^4(b - a)}{2(1 - \mu^2)m}\right] \tag{21}$$

Equations (16) to (19) can be modified to apply to this case if V_o^2 and the constant 1.8541 in each of these equations is replaced as follows: Since C is changed, replace V_o^2 by the bracketed term in equation (21); a new constant for the time of arrest (to replace 1.8541) is determined from the tables by requiring that equation (16) satisfy $g_1(0) = \beta b$ instead of zero. Of course, this new constant depends on β.

CIRCULAR-MEMBRANE ANALYSIS

The second problem considered is a fixed circular membrane of radius b with a centrally located attached mass M of radius a. (See fig. 1(b).) The boundary conditions on displacements are zero deflection at $r = b$:

$$w(b,t) = 0 \tag{22}$$

and zero in-plane displacement at $r = a$ and $r = b$:

$$\left.\begin{aligned} u(a,t) &= 0 \\ u(b,t) &= 0 \end{aligned}\right\} \tag{23}$$

The boundary condition on stress resultant at the mass (at $r = a$) is:

$$N_r(a,t)w_{,r}(a,t) = \frac{M}{2\pi a} w_{,tt}(a,t) \tag{24}$$

Initially the circular membrane-mass system is assumed to have zero lateral deflection and some characteristic speed distribution with the mass moving at speed V_o. Thus the initial conditions are

$$w(r,0) = 0 \tag{25}$$

and

$$w_{,t}(r,0) = f(r) \tag{26}$$

The characteristic speed distribution $f(r)$ must be equal to zero at $r = b$ and equal to V_o at $r = a$.

The equation of equilibrium of the radial forces in the circular membrane is

$$N_{r,r} + \frac{1}{r}\left(N_r - N_\theta\right) = 0 \tag{27}$$

The stress resultants N_r and N_θ are given in terms of displacement by

$$N_r = \frac{Eh}{1 - \mu^2}\left(u_{,r} + \frac{1}{2}w_{,r}^2 + \mu\frac{u}{r}\right) \tag{28}$$

$$N_\theta = \frac{Eh}{1 - \mu^2}\left(\frac{u}{r} + \mu u_{,r} + \frac{\mu}{2}w_{,r}^2\right) \tag{29}$$

which, upon substitution into equation (27), lead to the following equilibrium equation:

$$u_{,rr} + \frac{1}{r}u_{,r} - \frac{1}{r^2}u + w_{,rr}w_{,r} + \frac{1 - \mu}{2r}w_{,r}^2 = 0 \tag{30}$$

With the mass of the membrane considered negligible in comparison with the attached mass, the equation of equilibrium of the transverse forces is

$$\frac{1}{r}\left(rN_r w_{,r}\right)_{,r} = 0 \tag{31}$$

or

$$\left[r\left(u_{,r} + \frac{1}{2}w_{,r}^2 + \mu\frac{u}{r}\right)w_{,r}\right]_{,r} = 0 \tag{32}$$

If $\mu = \frac{1}{3}$ the solution for u is simply $u = 0$, and the solution for w is given by equation (32), which becomes

$$\left(rw_{,r}^3\right)_{,r} = 0$$

and therefore

$$w = g_1(t) + r^{2/3}g_2(t)$$

Boundary condition (22) leads to the relation

$$g_2(t) = -\frac{1}{b^{2/3}}g_1(t)$$

or

$$w = \left[1 - \left(\frac{r}{b}\right)^{2/3}\right] g_1(t) \tag{33}$$

It is expected that the results are not strongly affected by the value of μ, and since 1/3 is a practical value of μ for many materials, the remainder of the analysis is limited to this case.

The form of equation (33) suggests the possibility that this problem for $\mu \neq \frac{1}{3}$ as well as $\mu = \frac{1}{3}$ may be solved by initially assuming separation of variables. A replacement for the r-dependent factor in equation (33) can be determined for $\mu \neq \frac{1}{3}$. It is in the form of the corresponding classical static loading solution $\left(\text{which is given by considerably more complicated relations for } \mu \neq \frac{1}{3}\right)$.

From equation (33) and the conditions specified, the characteristic speed distribution is

$$f(r) = V_o \frac{b^{2/3} - r^{2/3}}{b^{2/3} - a^{2/3}} \tag{34}$$

Substitution of equation (33) into equations (24), (25), and (26) gives the following nonlinear differential equation and initial conditions from which g_1 can be found:

$$\ddot{g}_1 + \frac{8\pi Eh}{27b^2\left[1 - \left(\frac{a}{b}\right)^{2/3}\right]M(1 - \mu^2)} g_1^3 = 0 \tag{35}$$

$$g_1(0) = 0 \tag{36}$$

$$\dot{g}_1(0) = V_o \frac{b^{2/3}}{b^{2/3} - a^{2/3}} \tag{37}$$

Equations (35), (36), and (37) are identical in form to equations (12), (13), and (14) of the previous problem. The results that follow were obtained in the same way as those of the previous problem:

$$t_A = 1.8541\left[1 - \left(\frac{a}{b}\right)^{2/3}\right]^{3/4}\sqrt[4]{\frac{27\left(1 - \mu^2\right)Mb^2}{16\pi EhV_o^2}} \qquad (38)$$

$$\frac{w}{b} = \frac{1 - \left(\frac{r}{b}\right)^{2/3}}{\left[1 - \left(\frac{a}{b}\right)^{2/3}\right]^{1/4}}\sqrt[4]{\frac{27\left(1 - \mu^2\right)MV_o^2}{4\pi Ehb^2}}\ \mathrm{cn}\left[1.8541\left(1 - \frac{t}{t_A}\right),\frac{\sqrt{2}}{2}\right] \qquad (39)$$

$$N_r = \frac{\left(\frac{b}{r}\right)^{2/3}}{\left[1 - \left(\frac{a}{b}\right)^{2/3}\right]^{1/2}}\sqrt{\frac{EhMV_o^2}{3\pi\left(1 - \mu^2\right)b^2}}\ \mathrm{cn}^2\left[1.8541\left(1 - \frac{t}{t_A}\right),\frac{\sqrt{2}}{2}\right] \qquad (40)$$

The results given in equations (38) to (40) are also presented in figures 2, 3, 7, 8, and 9.

DISCUSSION OF FIGURES

In the previous sections equations giving the deflection and stresses were obtained in closed form for the arrest of a moving mass attached to a membrane strip and to a circular membrane. In this section the figures presenting the results of these equations are discussed.

The time variation of deflection of the mass is presented in figure 2 and the time variation of stress resultant at the mass is presented in figure 3 for both the membrane-strip and the circular-membrane problem. Both figures show variations for the first quarter-period of vibration - that is, from the initial zero-deflection position to the time of arrest.

The time of arrest t_A is given for the membrane strip in figure 4 and for the circular membrane in figure 7 as a function of a/b, the ratio of comparable dimensions of the mass and the membrane. (See fig. 1.) Although the parameters are somewhat different, the trends are roughly the same.

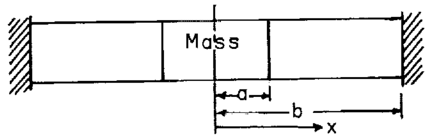
The solid lines in figures 5 and 8 give the deflection as a function of distance from the center for several values of a/b. The dashed line gives the maximum deflection (deflection of the mass) for all values of a/b. The trends for the strip and circular membrane are again roughly the same. The solid lines in figures 6 and 9 give the stress-resultant distribution; the dashed line gives the stress resultant at the mass. The stress-resultant distribution is constant for the strip, and the stress resultant at the mass increases with increase in relative size (dimension) of mass. For the circular membrane the stress resultant at the mass first decreases and then increases with increase in relative size of mass. For a mass of large dimensions the results for the strip and the circular membrane are essentially the same.

CONCLUDING REMARKS

The analysis is presented for the symmetrical motion of a membrane strip with a centrally located attached mass initially moving at a given speed. Results are presented for the initially flat membrane and for the case in which the membrane is slack until a given slope is reached.

The analysis and results are presented for the symmetrical motion of a circular membrane having a Poisson's ratio of 1/3 with a centrally located attached mass initially moving at a given speed. These results, as well as results for other values of Poisson's ratio, may be obtained by the method of separation of variables. The present results are for a practical value of Poisson's ratio (1/3) and the relations involved are considerably less complicated than for other values of Poisson's ratio.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., May 1, 1963.



(a) Membrane strip.

(b) Circular membrane.

Figure 1.- Dimensions of configurations studied.

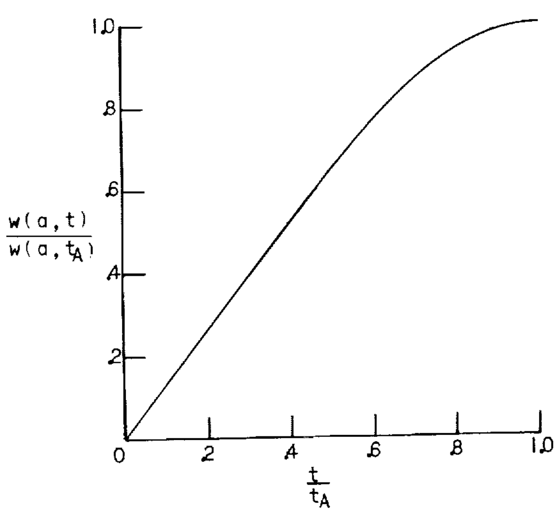


Figure 2.- The time variation of deflection of the mass from the initial zero-deflection position to the time of arrest t_A for both the membrane strip and the circular membrane.

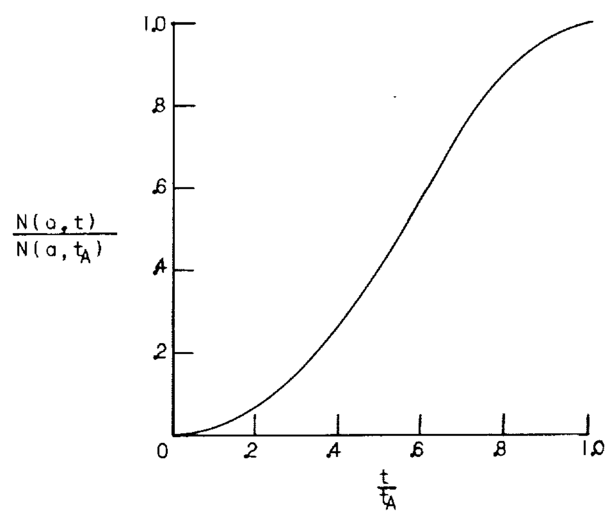


Figure 3.- The time variation of stress resultant in the membrane at the mass from the initial zero-deflection position to the time of arrest t_A for both the membrane strip and the circular membrane.

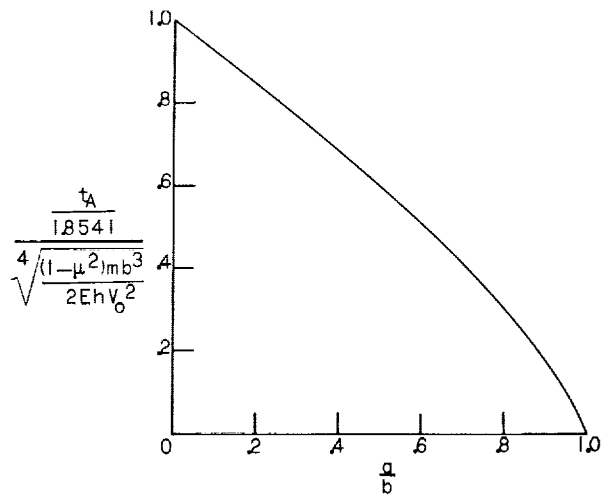


Figure 4.- The time of arrest of a mass attached to a membrane strip.

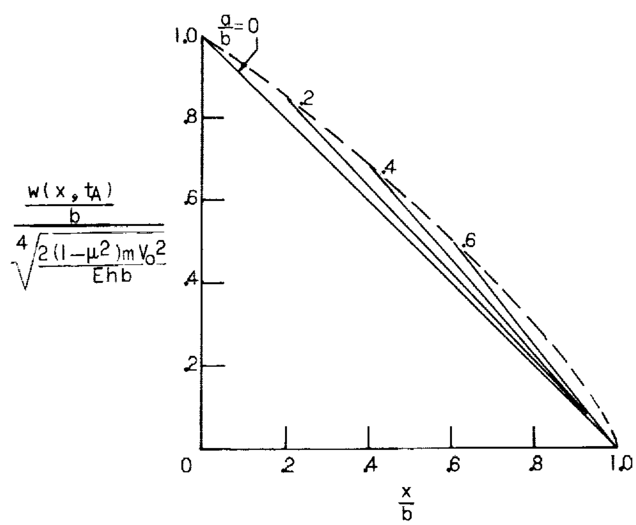


Figure 5.- Distance variation of deflection for membrane strip.

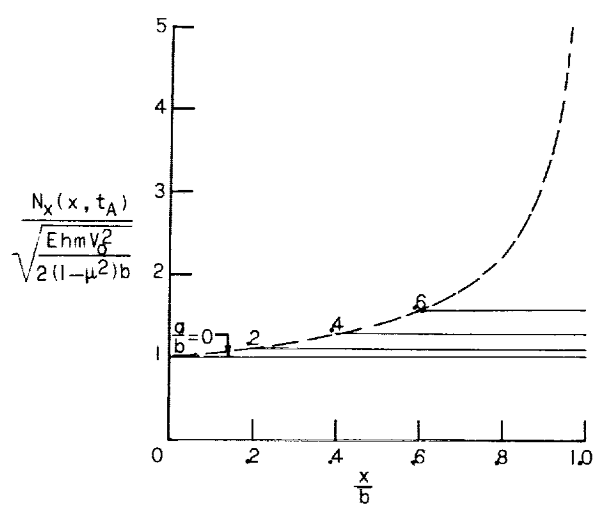


Figure 6.- Distance variation of stress resultant for membrane strip.

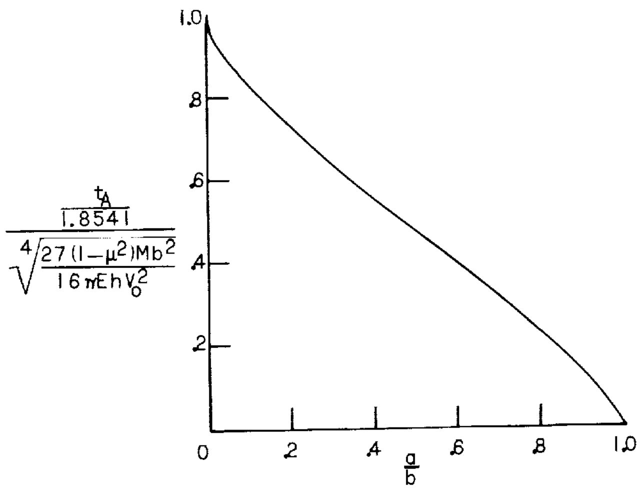


Figure 7.- The time of arrest of a mass attached to a circular membrane.

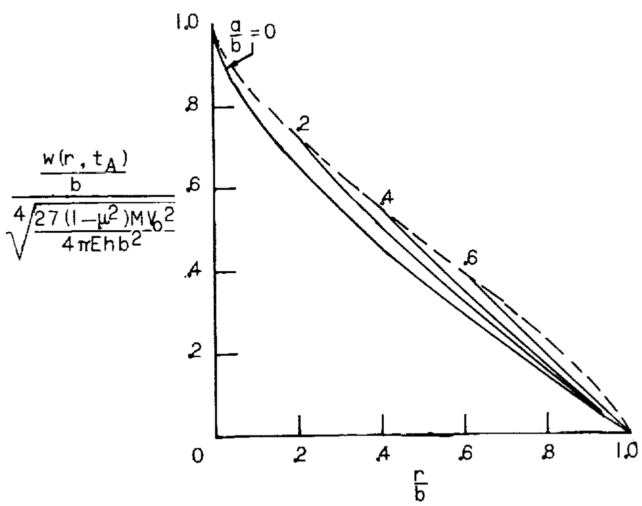


Figure 8.- Radial distance variation of deflection for circular membrane.

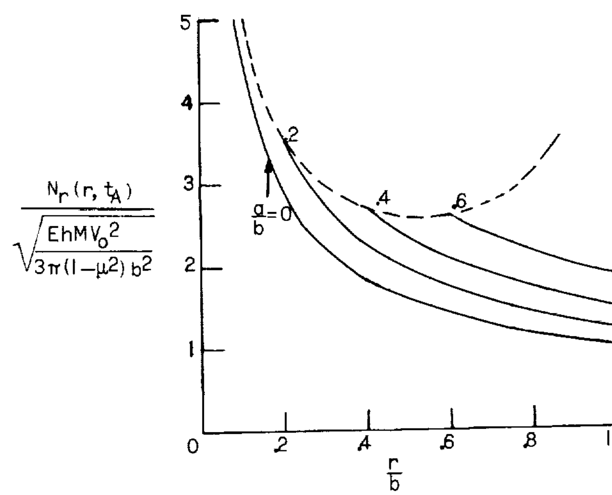


Figure 9.- Radial distance variation of radial stress resultant for circular membrane.

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